Thesis subject 2019

Laboratory: Institut Jean Le Rond d'Alembert - Equipe MISES
University: Sorbonne Université - CNRS UMR 7190

Title of the thesis: Extreme periodic modulation of elastic states: enhanced parametric instabilities for functionalities.

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Collaborations within the thesis:

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This subject can be published on the doctoral school's web site: YES

Thesis's summary (abstract):

Parametric instabilities, caused by the periodic modulation of an elastic structural state, can be used to fulfill engineering dynamical functionalities. Currently though, the benefit of these elastic instabilities is mostly restricted to certain MicroElectroMechanical resonating Systems (MEMS) in which damping is low enough for these instabilities to exist and be controlled. Here, instead of seeking for ways of lowering friction forces, we propose to drastically modulate mechanical states in order to enhance parametric instabilities and therefore allow for new promising dynamic functionalities whatever the damping coefficients of a structure.

The aim of this thesis is to design and study extreme parametric oscillators, which have been overlooked nonlinear dynamical systems whose energy landscapes are periodically varied in time. First, by mean of theoretical developments and the realization of fundamental pendulum experiments, the candidate will strive to identify the main geometrical and mechanical parameters, or design rules, that would lead to interesting dynamical features in extreme parametric oscillators such as optimal energy transfers or discrete frequency filtering. Those developed concepts would then be applied for the design of a vibratory elastic beam or plate that would constitute a first proof of concept of extreme parametric modulation in structures, which could pave the way for an eventual future implementation in MEMS.
Extreme periodic modulation of elastic states: enhanced parametric instabilities for functionality

**Context:** Nowadays, from the use of buckling for folding [1,2] (see Fig.1a) to the exploitation of fluttering piezoelectric flags for energy harvesting [3] (see Fig.1b) or the benefit of parametric resonances for the reduction of parasitic signals in MicroElectroMechanical Systems (MEMS) [4] (see Fig.1c), elastic instabilities eventually occurring in slender elastic structures are often seen as an opportunity to seize rather than a failure to avoid. Parametric elastic instabilities, found in many engineering problems [5-6], are caused by the self-synchronized periodic modulations of the elastic state of a slender structure [7]. Although promising for elaborate functionalities, the optimal use of parametric instabilities in elastic structures is often restricted to MEMS devices where damping is sufficiently low for the principal and subharmonic instability regions to exist [8,9]. To overcome this drawback and fully exploit the potential of parametric instabilities for functionality at any scales, a change of paradigm is necessary. Instead of classically lowering frictions to favor parametric excitations from the small modulations of an elastic state, one could periodically impose a drastic change of elastic state to enhance dynamical instabilities at common damping, as explained by the preliminary experiment and sketches in Fig.2.

*Fig. 1.* Different examples of elastic instabilities for engineering functionalities. a) The collaborative local buckling triggered on the circumference of a thick elastomeric cylinder allows for its global twisting motion. This mechanism could be used as a building block for soft robotics [2]. b) It is possible to harvest the energy of an incoming flow from the fluttering instability of a piezoelectric flag [3]. c) The SEM image shows the first MEMS device that exhibited a fifth subharmonic parametric resonance in 1998 [4]. In 2016, a research group exploited twenty-eight orders of parametric resonances in MEMS for multi-band vibration energy harvesting [10].

**Scientific objective and expected results:** The goal of the thesis is to characterize, theoretically and experimentally, the mechanics of the extreme periodic modulation of elastic states in order to establish basic design rules for new functionalities in slender elastic structures. This particular vibrational phenomenon has been overlooked in the literature and the design of original experimental proofs of concept will be required. Also, because the periodic change of elastic states is qualitatively important (see Fig.2), analytical tools associated with classic parametric oscillators [11,12] (multiple scale, averaging, spectral methods...) will not be applicable and alternative mathematical approaches of dynamical systems will be needed for the elaboration of theoretical models [13,14]. The rich spectral nature of parametric instabilities enhanced by the extreme modulation should offer promising opportunities for dynamical properties of structures. The capacity to trigger high subharmonic instability regions will favor the energy transfers between the parametric excitation and the elastic state, leading to efficient possibilities in large-band energy harvesting or...
energy absorption [15]. Also, as shown in Fig.2c, two-state modulation [16] should allow for optimal passive dynamical stabilization of naturally diverging systems (which is not to be confused with dynamical stabilization based on feedback nonlinear control [17]), a concept that could lead to almost discrete frequency and amplitude filters.

![Fig. 2. Example of an extremely modulated magneto-mechanical state. a) Picture of a preliminary experiment: a pendulum whose mass is made of steel, is symmetrically placed between two identical attracting electromagnets that are periodically turned on (red energy states in b-c)) and off (blue energy states in b-c)). Because the symmetric energy landscape varies drastically, parametric instabilities are enhanced. b) Even with low and slow periodic electromagnetic forces, highly parametric instability regions are easily reached which allow for large-band energy harvesting. c) Even if the electromagnetic forces are strong most of the time, it should exist an optimal modulation function for which the naturally diverging mass is dynamically stabilized in its central position, a property that can lead to discrete spectral filtering. Here, we used a pendulum and electromagnets to perform two-states modulation but one could as well use a slender elastic structure and large compressive loads for example.](image)

**Detailed program:** The candidate is expected to have a solid background in dynamical systems as well as a deep inclination in understanding the physics behind structural mechanical problems. Starting from the work explained in [7,16,18] on the dynamics and stability of structures in periodic elastic states, the candidate will progressively seize the delicate concepts of parametric instabilities on theoretical and experimental problems with increasing difficulties in order to build design rules for engineering functionalities based on extreme modulation of periodic states. A detailed program would be the following:

- The first step will be to develop and understand the concepts and methodologies (Floquet forms, frequency lock-in, nonlinear limit cycles, ...) associated with the theoretical and experimental analysis of the fundamental example of the magneto-mechanical proof of concept illustrated in Fig. 2a.

- The candidate will then need to develop mathematical and numerical models to identify the main mechanical and geometrical features responsible for the enhancement of parametric instabilities regarding three promising dynamical properties for functionalities:
  - The opportunity for vibrations absorption in the nonlinear regime of parametric oscillators whose elastic state is diverging on a short-period of time.
  - The feasibility for large-band energy harvesting by finding the configurations that allow for very high subharmonic instability regions to exist.
  - The possibility of optimal dynamical stabilization and frequency filtering by seeking the modulation functions with the minimum power necessary to stabilize a naturally diverging system.

- Finally, the candidate will extend the previous concepts to the nonlinear dynamics of slender elastic structures in order to be able to import the ideas of enhanced parametric instabilities in high-frequency vibration systems such as MEMS. Because of the geometric nonlinearities at place in those systems, the candidate will need to gain some profound theoretical understanding on the dynamics of slender elastic structures [19,20]. Also, part of the challenge for the candidate will be to propose and design original enhanced parametric elastic oscillators for example based on time-periodically buckling structures.
REFERENCES:


