ED SMAER

Sujet de thèses 2014

Laboratoire : Institut Jean Le Rond d'Alembert - Equipe MISES

Etablissement de rattachement : CNRS UMR 7190 - Université Pierre et Marie Curie Paris

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Codirection et section CNU ou CNRS : Arnaud Lazarus (MCF, CNU 60)

Titre de la thèse :
Fr : Analyse modale de systèmes mécaniques en états périodiques.
En: Modal analysis of mechanical systems in periodic states.

Collaborations dans le cadre de la thèse :
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Rattachement à un programme :

Le sujet peut être publié sur le site web de l'ED SMAER : OUI .

Résumé du sujet :

L'analyse modale, très utilisée en ingénierie des structures, consiste à récupérer les informations intrinsèques physiques d'un système mécanique en équilibre en le perturbant au premier ordre. Floquet a depuis longtemps montré qu'il était toujours possible, en théorie, d'utiliser une approche modale pour caractériser les systèmes mécaniques dynamiques, dont les propriétés physiques varient périodiquement, comme dans le cas de nombreuses structures en rotation ou vibrations. Pourtant, il n'existe pas encore aujourd'hui d'analyse modale de structures en états périodiques malgré la prolifération de ces types de problèmes en mécanique, la principale raison étant la difficulté à calculer numériquement les propriétés spectrales de grands systèmes d'équations différentielles linéaires à coefficients périodiques.

L'objectif de la thèse sera de mettre en place les outils numériques et expérimentaux pour l'analyse modale de systèmes mécaniques en états périodiques. Une fois la méthodologie mise en place, il s'agira de montrer l'intérêt des modes de Floquet calculés dans différents domaines de la mécanique des structures élastiques élancées. A l'image de l'analyse modale classique, les modes de Floquet permettront de procéder à l'identification modale de structures en état périodiques. Ces modes devront aussi faciliter le calcul et notre compréhension physique de la stabilité d'orbites périodiques de systèmes mécaniques. Enfin, l'utilisation de l'analyse modale pourrait aussi être une alternative aux ondes de Bloch dans les mileux périodiques pour la caractérisation de métamatériaux.

Le candidat, doté d'une culture solide en calcul de structures, devra également être intéressé par la compréhension physique des problèmes mécaniques traités.

Sujet développé (à présenter en 2 ou 3 pages maximum,
en précisant notamment le contexte, les objectifs, les résultats attendus)

Context: Modal analysis, based on the spectral decomposition of linear discrete dynamical systems, is a powerful and widely used method in mechanical engineering for identification in structural vibration [1], as illustrated in Fig. 1a, determination of local stability of equilibrium states [2,3] or model reduction techniques [4]. The classic concept of structural mode however, is only valid for structures in equilibrium, i.e. where the geometrical and mechanical characteristics of the system can be considered constant with time. According to Floquet theory however [5], it should still be possible to compute modes, denoted Floquet Forms (FFs), of mechanical systems periodically varying with time. This larger class of systems is of practical interest since it covers mechanical systems such as asymmetric rotating machines as shown in Fig. 1b, axially oscillating systems depicted in Fig. 1c, structures vibrating with large amplitudes or flute-like musical instruments. Although many numerical techniques based on Floquet's theory have been developed [6-10], FFs are not computed and used, mostly due to the difficulty to achieve spectral decomposition of large linear time-varying discrete dynamical systems, resulting in a loss of modal informations [11].

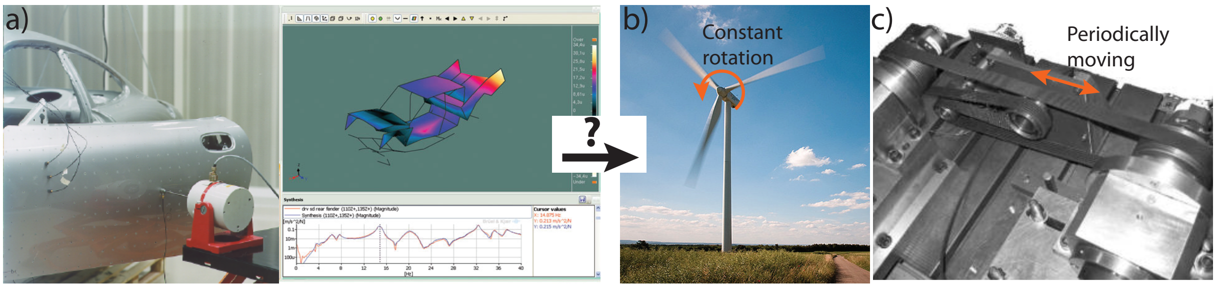

Fig. 2. Various examples of structural systems in equilibrium or periodic states. a) Experimental and numerical modal analysis performed on a car. b) The vibration of a wind turbine in rotation is an example of periodically time-varying system. c) The dynamical response of an industrial axially moving belt is another example of such a class of problems (courtesy of G. Michon et al.).

Scientific objective and expected results: The goal of the thesis is to generalize the tools of classic modal analysis of structures in equilibrium to structures in periodic states. The key point for success will be to numerically achieve the spectral decomposition of large periodically time-varying linear dynamical systems in order to extract the Floquet forms carrying modal informations. The ability to efficiently compute high dimensional FFs should impact numerous aspect of structural mechanics. Like in classic modal analysis, computation of FFs could allow for modal identification of structures in periodic states as shown in Fig. 1b-c. Similar to the role of eigenvalues and eigenmodes in quasi-static stability analysis, FFs could also provide physical insights in the stability analysis of systems in periodic states such as slender structures in nonlinear vibrations. Also, due to their intermediary position between classic linear normal modes and more complex nonlinear normal modes [12], they could be interesting candidates for reduced order modeling in structural vibration. Finally, computation of FFs could have applications in the stability and homogenization of spatially periodic structures and offer a challenging alternative to Bloch wave analysis [13].

Detailed program: The candidate is expected to have a solid background in computational mechanics as well as a deep inclination in understanding the physics behind mechanical problems. Starting from the preliminary work found in [4,14,15] on the linear stability of periodic orbits, the candidate will progressively seize the delicate concepts of Floquet Forms on problems with increasing difficulties in order to build a rigorous framework for modal analysis of structures in periodic states. A detailed program would be the following:

- The first step will be to develop and understand the concepts and methodologies of spectral analysis of periodically-varying Ordinary Differential Equations through the archetypal example of the oscillations of a simple parametric pendulum illustrated in Fig. 2a. The obtained numerical results will be compared with more classic non-modal techniques derived from Floquet's theory such as Bloch wave like analysis [13], spectral computation of the monodromy matrix [7] or more recent approaches from the litterature [9].

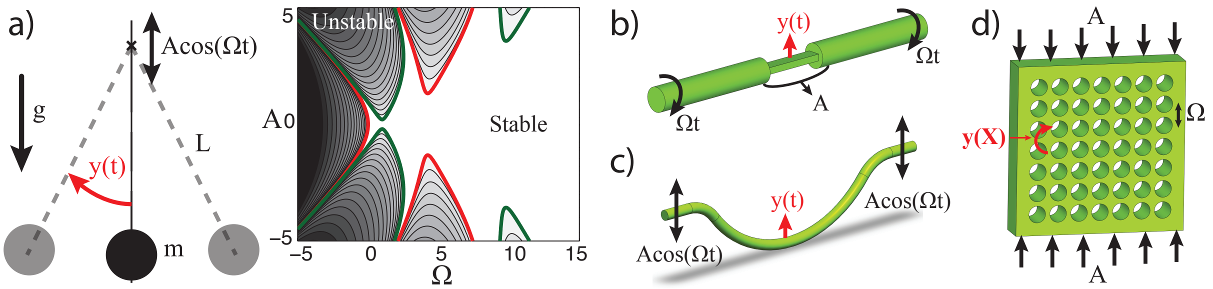

Fig. 2. Mechanics of slender elastic structures in periodic states. a) The parametric pendulum. Linear oscillations, y(t), can be expressed as a linear combination of FFs whose growth rate inform on the stability of the axially moving pendulum. b) Oscillations of an asymmetric rotor can be described with FFs. c) The linear stability of nonlinear vibration of a slender beam is determined by its FFs. d) The Bloch wave of a compressed periodic elastomeric structure can be related to FFs in 2 or 3-dimensions.

- To extend the numerical method to practical structural systems with a large number of degrees of freedom, the following step will be to incorporate the concepts developed on the parametric pendulum in a more general numerical framework such as the Finite Element Method (potentially in Cast3m [16], matlab or fortran). This generalization step should be quite direct but could required significant work if reduction of time computation is required.

- The original numerical method will then be challenged and validated by performing experimental modal identification of axially vibrating elastic beams and geometrically asymmetric elastic rotors (Fig. 2b) which are both common examples of structures in periodic states. The originality of the model experiments will be to use digital fabrication techniques such as laser cutting and 3D printing to accurately controled the geometry of our samples [17]. Another innovation from this work will be the development of experimental techniques to properly observe and measure Floquet Forms.

- A second application for FFs will be the determination of orbital stability of the nonlinear vibration of slender structures as illustrated in Fig. 2c. Like classic modal analysis, the interest of FFs is that it gives access to linear stability as well as tangential direction of perturbation, but for periodic orbits instead of equilibrium states as for linear normal modes. One particular aim will be to seize the potential of FFs in the continuation of periodic orbits, for example, as relevant tangential directions of a prediction step of a Newton's algortihm. To avoid spending too much time on the technical development of a continuation method from scratch, an existing code such as MANlab [18] can be used. Again, model experiments will be performed on the nonlinear vibrations of beams in order to validate the computations.

- Finally, according to the timeline, some work could be done to explore the analogy between temporal and spatial periodicity. The idea will be to use FFs for the stability and mechanical response of spatially periodic structures such as mechanically tunable metamaterial [19] whose model experiments is represented in Fig. 2d. In practice, the goal will be to explore the potential interest of FFs over classic Bloch wave analysis. As always, to emphasize our numerical results with physical insights and reality, some experiments will be performed.

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